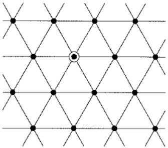
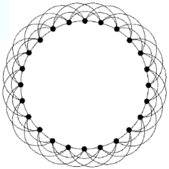
Small-World models

- Apart from heavy-tailed degree distributions, real-world networks show many other interesting properties
- High amount of transitivity and low value of average path length usually coexist

Transitivity vs Low path lengths

- Transitivity remains a poorly understood property
- Many network models can be constructed that have high clustering
  Triangular lattice, Non-locally coupled ring
- However, such networks are "large-worlds"
- Random networks like ER graph have very small path lengths





Watts-Strogatz model

- Start with a regular lattice with high clustering
- Randomize this network by rewiring each link with probability p
- When $p = 0$, no edges are rewired, and the original circle is retained.
- When $p = 1$, all edges are rewired, and we get back the random graph
- The model shows interesting behavior for the intermediate values of p

Collective dynamics of 'small-world' networks

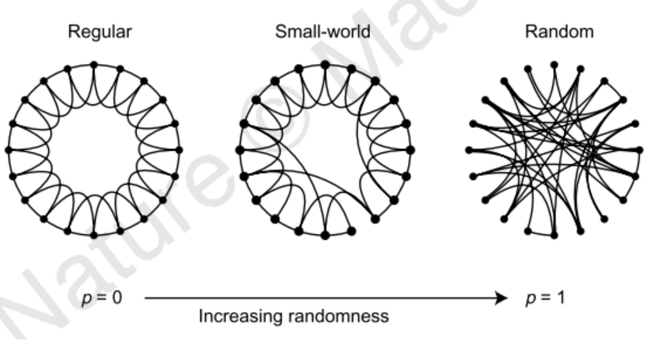
Duncan J. Watts* **& Steven H. Strogatz**

Department of Theoretical and Applied Mechanics, Kimball Hall, Cornell University, Ithaca, New York 14853, USA

Networks of coupled dynamical systems have been used to model biological oscillators[1–4], Josephson junction arrays[5,6], excitable media[7], neural networks[8–10], spatial games[11], genetic control networks[12] and many other self-organizing systems. Ordinarily, the connection topology is assumed to be either completely regular or completely random. But many biological, technological and social networks lie somewhere between these two extremes. Here we explore simple models of networks that can be tuned through this middle ground: regular networks 'rewired' to introduce increasing amounts of disorder. We find that these systems can be highly clustered, like regular lattices, yet have small characteristic path lengths, like random graphs. We call them 'small-world' networks, by analogy with the small-world phenomenon[13,14] (popularly known as six degrees of separation[15]). The neural network of the worm *Caenorhabditis elegans*, the power grid of the western United States, and the collaboration graph of film actors are shown to be small-world networks. Models of dynamical systems with small-world coupling display enhanced signal-propagation speed, computational power, and synchronizability. In particular, infectious diseases spread more easily in small-world networks than in regular lattices.

Publishers Ltd 1998

Watts-Strogatz model



Watts-Strogatz model

- The model shows that transitivity and low average path length can coexist
- Small path lengths develop in the model even for very small values of p. This provides explanation of why almost all real networks have low average path lengths
- The model is not analytically tractable
- A variant where only edges are added is much easier to analyze